Green Shoots? Where, when and how?*

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Abstract

What is the meaning of green shoots? In this paper we provide a statistical definition of this term which allows us to analyze where, when and how the recovery started. With the same methodology, we confirm that there are symptoms of recovery in the US, the Euro area and Spain with some differences in timing. In addition, we find some leading behavior from the quotes in the press to the actual confirmation of the data, even when the data include variables with clear expectations contents.

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1 Introduction

In the middle of the current recession, analysts, policy makers, and journalists, have used the term green shoots to refer to signals of the end of the recession period. Although this expression was first used in this sense by Norman Lamont, the Chancellor of the Exchequer of the United Kingdom, during the 1991 recession, it was popularized in US by Ben Bernanke, chairman of the Federal Reserve Board, who ensured on March 15th 2009 that he detected green shoots of economic recovery. From this quote, the use of the expression has been massive, with more than 189 million entries in Google since then.

Needless is to say that the term green shoots has not always been used with scientific criteria mainly for two reasons. First, the term is very imprecise so it leaves the users of the term to identify where, when and how the recovery comes depending basically on their own believes. Of course, the signals of recoveries do not appear in all the economic indicators with the same intensity at the same time in different countries. Hence, the skeptical users will be inclined to accentuate the negative signals of some indicators while the optimistic users will be tempted to stress the positive signals of some others. Perhaps, it is also the impreciseness of the definition of green shoots what also diminishes the meaning of international comparisons of the existence of these green shoots. Second, in the search of green shoots, the recent advances in information technologies make the number of variables with information about the economy exponentially growing and with an unprecedented updating frequency. The cost of checking in real time the publication calendar of the variables, the latest release and the revisions makes very difficult the task of the analyst of keeping updated to check every day if the shoots are actually green.

The purpose of this paper is to provide the economic agents with a statistical definition of the term green shoots which is very easy to interpret for the general public. In particular, we will define green shoots as a low probability of being in a recession on period $t$ with the information available up to that period. This definition overcomes the two problems previously stated associated with the increasingly use of the term green shoots. First, probability of recession is no longer an imprecise term. The inferences about the state of the cycle are computed from a statistical model applied to data which is then transparent
and objective. In addition, since recession probabilities are free of units of measurement, international comparisons are easily allowed. Second, if the probability of recession is computed based on a set of variables that the agents consider as representative of economic activity, because the chosen variables are good proxies of the general economic activity, this probability of recession should be a "sufficient statistic" for the analysts with the subsequent saving of time and cost for them. We pretend, using a computationally simple algorithm, to compute an easy to interpret number that the users can update when needed.

To compute the probabilities, we propose a dynamic factor model from a set of economic indicators that captures expansion and recession phases as unobserved regime shifts in the mean of the common factor. The unobserved state variable controlling the regime shifts is modeled as following a Markov process as in Hamilton (1989) but differs from other methods employed in the literature in a number of important ways. First, most of the empirical applications fit the Markov switching process to GDP series, assuming that this variable captures all the relevant information about the state of economy. However, the NBER dating committee defines a recession as a significant decline in economic activity spread across the economy normally visible in production, employment, real income, and other indicators. In addition, although these models usually find good in-sample accuracy to identify business cycles, the time delay in the publication of GDP data makes them useless when addressing the current situation of the economy in real time. For example, on the date in which this paper is written (October 4th 2009) the latest available US GDP release is for the second quarter of 2009. When making a statement about the probability of being in a recession today, one need to calculate the probability from the latest observation available (2009.Q2) and with this information, to forecast the probability of being in a recession in the current quarter (2009.Q4). This implies missing extremely valuable information from June to October.

Second, following the lines of Diebold and Rudebusch (1996), some recent proposals (Kim and Yoo, 1995, Chauvet, 1998, and Kim and Nelson, 1998) estimate different versions of Markov-switching dynamic factor models which capture the notions of both co-movements across business cycle monthly indicators and regime switching. The indicators used by these proposals build on the tradition of the linear single-index dynamic factor
model of Stock and Watson (1991): following the logic of national accounting that GDP is approximated from the income side, the supply side and the demand side, they choose Industrial Production Index (supply side), total sales (demand side), real personal income (income side) and they add an employment variable to capture the idea that productivity does not change dramatically from one period to the other. Although the inference about the business cycle states from these proposals has been very accurate, there were still two important gaps. The first gap is the lack of theoretical results about the gains of these methods with respect to simpler alternatives. Do the multivariate models have better properties than the univariate specifications? Is it better to estimate nonlinear dynamic factor models or to estimate linear dynamic factor models and to compute business cycle inferences from the estimated common factor? These questions are unsolved in those previous papers. The second gap of these proposals is the restriction of handling only with complete data sets which implies that the models are not able to deal with some typical problems associated to real time forecasting such as mixing frequencies and ragged ends.

Our companion paper, Camacho, Perez Quiros and Poncela (2009) fill out these two gaps. We proved that, as long as the variables included in the multivariate framework are not very noisy, are synchronized with the business cycle and the idiosyncratic cross-correlation is not too high, there are significant in-sample and out-of-sample gains in estimating Markov-switching dynamic factor models versus univariate representations in terms of reduction of the mean squared error and mean squared forecasting error. Obviously these gains add up to the timely information available for the indicators versus the long delay in the information about GDP. In addition, our paper shows how to infer the state of the economy even when we do not have a full balanced panel which is a nontrivial feature since it allows us to handle with ragged ends and mixing monthly and quarterly information in a non linear framework. As far as we know, we are the first ones in enhancing these non linear multivariate dynamic models and we think that with these powerful econometric techniques we are prepared to technically handle the definition of green shoots, and the timing, the intensity and the form of the recovery.

Overall, our results suggest that the Markov-switching dynamic factor model is a potentially very useful filter to transform the information of a broad set of economic
indicators into recession probabilities. We provide some formal evidence regarding the speed with which the real-time monitoring can identify the latest turning point in US, the Euro area and Spain. As we expected, our model confirm the presence of green shoots in US, the Euro area and Spain with different timing in different economies. In addition, we confirm some leading time from the number of quotes in the press to the actual confirmation of the data, even when the data include variables with clear expectations contents.

The paper is structured as follows. Section 2 describes the results of standard univariate analysis using GDP. Section 3 discusses the multivariate extension in a small scale model with no expectation variables. Section 4 analyzes the results of a medium scale model with expectation variables. Section 5 looks at the real time analysis of the forecasts. Section 6 concludes.

2 Univariate analysis

Hamilton (1989) originally proposed an excellent framework to deal with business cycles in time series. Let us assume that the expected value of a time series, $y_t$, switch between two separate business cycle states which are usually known as expansions and recessions. In mathematical notation, it is assumed that $E(y_t) = \mu_0$ if the economy is in an expansion, and that $E(y_t) = \mu_1$ if the economy is in a recession. In his seminal proposal, Hamilton (1989) assumes that the time series is GDP and that apart from the transitions between expansions and recessions the series exhibits autoregressive dynamics. His econometric specification becomes

$$y_t = \mu_{s_t} + u_t,$$

where $u_t$ follows an AR(4) process, and $s_t$ is an unobservable state variable that takes on the value of 0 in expansions and the value of 1 in recessions. The dynamics of the state variable is supposed to follow a Markov chain of order one, which implies that

$$p(s_t = j|s_{t-1} = i, s_{t-2} = h, ..., I_{t-1}) = p(s_t = j|s_{t-1} = i) = p_{ij}$$

where $i, j = 0, 1$, and $I_t$ is the information set up to period $t$. In the related literature, it is standard to call $p(s_t = 0|s_{t-1} = 0) = p_{00}$, and $p(s_t = 1|s_{t-1} = 1) = p_{11}$. 

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Camacho and Perez Quiros (2007) have recently shown that when appropriately accounting for the different switches between regime dependent means, then the serial correlation that characterizes the regime switches is substituting for the serial correlation that would normally be modeled via autoregressive structures. Accordingly, models that accurately capture the sequence of recessions and expansions are dynamically complete and no additional autoregressive parameters are required to capture the dynamics of the series. In these cases, the model proposed will be

\[ y_t = \mu_{s_t} + \epsilon_t, \]

where \( \epsilon_t \) is an uncorrelated sequence of Gaussian errors with zero mean and variance \( \sigma^2 \).

Figure 1 plots the growth rate of each of the three economies for the longest available sample (downloaded on October 4th 2009) along with the shaded areas that refer to the NBER-designated recessions. In the US, the data covers from 1953.1 to 2009.2 and for the Euro area from 1991.1 to 2009.2. In the case of Spain, using the latest data set published by the National Statistical Institute, we have data from 1974.1 to 2009.2. As assumed by the simple univariate Markov switching model the GDP series exhibit negative growth rates within most of the NBER recessions. Figure 2 shows the in sample filtered probabilities of being in recessions as estimated in the US economy, in the Euro area economy and in the Spanish economy from the model in (3). As depicted in the US figure, using just GDP data alone without any reference to what NBER may have said, we would come up with a very similar conceptual scheme to the one that the NBER have traditionally relied on. In addition, with the latest available information, the probability that these economies be in recessions is still very high.

The maximum likelihood coefficient estimates of model (3), which are reported in Table 1, reveal some interesting results. First, regarding the sample period and the economic area considered, including the recent recession (left hand panel) leads to positive means in state \( s_t = 0 \) and negative means in the regime represented by \( s_t = 1 \). Accordingly, we can associate the first regime with expansions and the second regime with recessions. Second, overall expansions are more persistent than recessions since the estimates of \( p_{00} \) are higher than those of \( p_{11} \). The international comparison reveals that although recessions seems to
be more persistent in the case of the Euro area and Spain, it seems that this result largely depend on the different samples used to obtain parameter estimates since the persistence of recessions in US becomes similar to that of the European cases when using comparable samples for US. Third, conditional on being in state $i$, one can derive the expected number of months that the business cycle phases prevail as $(1 - p_{ii})^{-1}$, and the expected amplitude of this state as $\mu_i (1 - p_{ii})^{-1}$. In US, the expected duration and amplitude of a typical recession are 16.67 quarters and 17%, while those figures fall in the case of recessions to 3.84 quarters and 1.61%. These estimates accord with the well-known fact that recessions are shorter and milder than expansions on average. To examine the extent to which the current recession is different from the previous ones, the right hand panel of Table 1 shows the estimates of the Markov switching parameters obtained from a sample which ends in late 2007. In this case, recession are expected to last 3.44 quarters and are expected to imply a loss of 1.17% which are close to the previous estimates. So far, this recession has lasted 7 quarters and which has implied a loss of 3.14% so it is actually being longer and harder than expected. Fourth, concerning international comparisons it is worth pointing out that in the Euro area recessions are expected to be longer (5.88 quarters) and deeper (losses of 4.52%) but this results is largely due to the short length in the European GDP. In the case of Spain, the expected recessions are the longest (8.33 quarters) but milder (loss of 0.66%).

The ability of univariate Markov switching models to compute inference of business cycles in real time deserves a final remark. The high commonality in switch times of probabilities and the US business cycle phases identified by NBER observed in Figure 2 gives the impression that the simple univariate Markov-switching models applied to GDP fit the business cycle extremely well. But the good in-sample results of this figure are somehow tricky in what it plots the filtered probabilities of being in recession for given quarter by using GDP growth rates up to that quarter which are obviously not available when computing inferences in real time. Since the GDP publication lag is about 45 days after the end of the respective quarter, the latest quarter for which inference can be computed in this way is that of the second quarter of 2009. To infer the probability of recession for the current fourth quarter of 2009 one need to forecast compute two-period
ahead forecasts of the probabilities. To analyze the effect of the large publication delay of GDP in business cycle inferences, Figure 3 plots the two-period ahead forecasts of the probabilities.\footnote{Note that this exercise does not account for the effect of data revisions which would amplify the deterioration of the in-sample identification of business cycles.} This figure allows us to put a question mark in the ability of univariate Markov-switching models to infer recession probabilities in real time: for all the recessions the signals to monitor the current business cycle developments are mild and come too late. Accordingly, the natural way to proceed seems to be adding economic monthly indicators which incorporate more timely information about the state of the business cycle.

3 Multivariate analysis

Enhancing the Markov-switching model of GDP to incorporate economic indicators can be helpful for two reasons. First, because they are published with shorter delay so they can incorporate more timely information. Second, because is they are synchronized with GDP, they could help to increase the signal of turning points. For this purpose, Kim and Yoo (1995), Chauvet (1998) and Kim and Nelson (1998) combined the dynamic-factor and Markov-switching frameworks to incorporate the two main characteristics of the business cycle indicators: comovements and asymmetries. However, their empirical proposals do not incorporate theoretical results to justify the need of dealing with more than one indicator as in the original proposal of Hamilton (1989). Concerning the role of the number of indicators in identifying the business cycles, let us summarize some of the results obtained in Camacho et al. (2009).\footnote{For further details, interested readers are referred to Camacho, Perez Quiros and Poncela (2009).}

3.1 Theoretical framework

Let us start with a single-index dynamic factor model whose common factor follows a Markov switching process. Let $y_t = (y_{1,t}, \ldots, y_{N,t})'$ be the vector of $N$ observed time series which is generated by a non-observed common factor, $f_t$, and $N$ specific or idiosyncratic
components

\[
\begin{align*}
  y_t & = \mathbf{A} f_t + \mathbf{u}_t \\
  N \times 1 & \quad N \times 1 \quad 1 \times 1 \quad N \times 1
\end{align*}
\]

(4)

where \( \mathbf{A} = (\lambda_1, \lambda_2, \ldots, \lambda_N)' \) is the factor loading matrix.\(^3\) The common factor follows a Markov switching autoregressive process with changing mean:

\[
  f_t = \mu_{s_t} + \frac{a_t}{\phi(B)},
\]

(5)

where \( B \) is the backshift operator and \( \phi(B) = 1 - \phi_1 B - \ldots - \phi_p B^p \). We also consider that the idiosyncratic components have the dynamic structure

\[
  \mathbf{F}(B) \quad \mathbf{u}_t = \epsilon_t \\
  N \times N \quad N \times 1 \quad N \times 1
\]

(6)

where \( \mathbf{F}(B) = diag(F_i(B)) \) is a diagonal matrix that collects the specific dynamics of each idiosyncratic shock, with \( F_i(B) = 1 - F_{i,1} B - \ldots - F_{i,p_i} B^{p_i}, i = 1, \ldots, N \), and \( \epsilon_t \) is multivariate zero mean white noise with diagonal covariance matrix \( \Sigma_\epsilon \).\(^4\) We also assume that \( s_t \) evolves according to an irreducible 2-state Markov chain whose transition probabilities are defined by (2).

The real time applications of this specification exhibit three major shortcomings. The first drawback has to do with the usefulness of including additional indicators which is usually known as the role of \( N \). To illustrate the role of \( N \) in identifying the business cycle, let us consider the simple example of comparing \( N = 1 \) versus \( N = 2 \), i.e., the analyst has two observed indicators, \( y_{1,t} \) and \( y_{2,t} \), and needs to decide if \( y_{2,t} \) will be included in the model. The key comparison is then between the filtered probabilities \( \text{prob}(s_t | y_{1,t}, y_{2,t}) \) and \( \text{prob}(s_t | y_{1,t}) \), which will be called multivariate and univariate probabilities, respectively.

Adding \( y_{2,t} \) will be useful to detect turning points if \( \text{prob}(s_t = 1 | y_{1,t}, y_{2,t}) > \text{prob}(s_t = 1 | y_{1,t}) \) when \( s_t = 1 \) (recessions) and \( \text{prob}(s_t = 1 | y_{1,t}, y_{2,t}) < \text{prob}(s_t = 1 | y_{1,t}) \) when \( s_t = 0 \) (expansions).

\(^3\)For simplicity, we assume contemporaneous relations between the common factor the economic indicators.

\(^4\)Note that the model allows for common shocks to the economy (\( a_t \)) as well as for specific shocks to each economic indicator (\( \epsilon_t \)).
Camacho et al (2009) show that in recessions,\(^5\) \[ \text{prob}(s_t = 1|y_{1,t}, y_{2,t}) > \text{prob}(s_t = 1|y_{1,t}) \]

if

\[
\frac{f(y_{2,t}|s_t = 1, y_{1,t})}{f(y_{2,t}|s_t = 0, y_{1,t})} > 1.
\] (7)

Taking natural logarithms and averaging out for all possible outcomes of \(y_{2,t}\), equation (7) can be transformed into

\[
\int \ln f(y_{2,t}|s_t = 1, y_{1,t}) f(y_{2,t}|s_t = 1, y_{1,t}) dy_{2,t} - \int \ln f(y_{2,t}|s_t = 0, y_{1,t}) f(y_{2,t}|s_t = 1, y_{1,t}) dy_{2,t}.
\] (8)

Rearranging, this expression becomes

\[
\int f(y_{2,t}|s_t = 1, y_{1,t}) \ln \frac{f(y_{2,t}|s_t = 1, y_{1,t})}{f(y_{2,t}|s_t = 0, y_{1,t})} dy_{2,t},
\] (9)

which is the change in the relative entropy of the density \(f(y_{2,t}|s_t = 0, y_{1,t})\) with respect to \(f(y_{2,t}|s_t = 1, y_{1,t})\). These authors show that including \(y_{2,t}\) helps in identifying the business cycle phases when this quantity is non negative, which occurs when \(\lambda_1\) and \(\lambda_2\) are of the same sign. In addition, they show some results that helps the users to examine when additional variables must be included in the model depending on the extent to which they are noisy, synchronized and available in real time.

Using Markov switching dynamic factor models in real time forecasting exhibits a second drawback. The vast majority of empirical applications has to handle with mixing frequencies, especially quarterly and monthly. Although Mariano and Murasawa (2003) proposed a way to combine different frequencies with linear Kalman filters which is becoming very popular, treatment of mixing frequencies is still unknown in nonlinear contexts. To overcome this problem by extending the Mariano-Murasawa proposal to allow for Markov switching dynamics. In particular, quarterly stocks data are converted in monthly observations by treating the data as observed the month that they are issued and as unobserved otherwise. By contrast, assuming that arithmetic means can be approximated by geometric means, quarter-on-quarter growth rates of flow quarterly data \((g_t)\) can be viewed as weighted averages of the monthly-on-monthly lagged growth rates \((z_t)\) of the monthly underlying series which are assumed to be known

\[
g_t = \frac{1}{3} z_t + \frac{2}{3} z_{t-1} + \frac{2}{3} z_{t-2} + \frac{2}{3} z_{t-3} + \frac{1}{3} z_{t-4}.
\] (10)

\(^5\)The treatment of expansions is symmetric.
For simplicity, let us assume that \( z_t \) refers to the first variable in \( y_t \) and that \( \phi(B) = 1 \). According to (4), the monthly underlying variable admits the decomposition into the sum of common and idiosyncratic components, \( z_t = \lambda_1 f_t + u_{1t} \). Since \( s_t \) takes two possible values, then \( z_t|s_t \) will follow the Gaussian distribution:

\[
f(z_t|s_t = i) = \frac{1}{\sqrt{2\pi \sigma^2}} \exp \left( -\frac{(z_t - \mu_i)^2}{2\sigma^2} \right),
\]

where \( i = 0, 1 \). Hence, \( f(z_t) \) is a mixture of two Gaussian distributions

\[
f(z_t) = \sum_{i=1}^{2} \pi_i f(z_t|s_t = i),
\]

where \( \pi_i \) is the unconditional probability of being in state \( i \). By equation (10), \( g_t \) is a linear combination of \( z_t \) and four lags. This implies that its density function is a mixture of \( 2^5 = 32 \) Gaussian distributions which can be computationally cumbersome in empirical analyses. However, Camacho et al (2009) show that accounting for just two states is enough to infer state probabilities.

The third drawback in real-time applications is handling with missing data which came from mixing frequencies and the ragged ends that characterize data publications. One potential solution could be skipping data from different frequencies and waiting until dealing with balanced data sets. However, this strategy will be very costly in the sense that it is very important to process the new information as soon as it arrives because the forecast probabilities will decay very fast and become uninformative very soon.\(^7\) In this context, Camacho et al (2009) show the benefits from including more promptly available indicators and how to handle missing observations in this non-linear context by extending the proposal by Mariano and Murasawa (2003). In particular, missing data are replaced by random draws \( \theta_t \) from \( N(0, \sigma^2) \) which must independent of the model parameters. The substitutions allow the matrices of the Kalman filter to be conformable but they have no more impacts on the model estimation than adding a constant in the likelihood function. This leads the forecasting procedure to become an extremely easy exercise. Computing

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\(^6\)For simplicity, we assume state-independent variances. However, allowing for state-dependent variances is straightforward.

\(^7\)Recall the empirical example of forecasting only from GDP.
3.2 Variables selection

According to the feasibility restrictions of nonlinear models, the number of variables that can be analyzed must be small and the selection of variables to be included in the analysis needs to be carefully done. However, note that the variable selection problem does not only affect small scale models. The standard linear large scale models never use all the time series available in real time at all levels of disaggregation for all the countries and regions used in the analysis. In addition, the level of complexity that large scale models incorporate to real time analysis is not always justified. In the context of forecasting, Boivin and Ng (2006) have recently suggested that, given the small number of categories that we have in macroeconomic data, the forecast accuracy does not necessarily increase with the number of series included in the model because these series might only add cross correlation in the idiosyncratic noise. Finally, Banbura and Runstler (2007) find that most of the predictive content of their large scale model is contained in a small set of variables.

Accordingly, we start from a simple model, following the suggestion of Stock and Watson (1991). Their idea follows the logic of national accounting that robust estimates of GDP are obtained by computing GDP from the income side, the supply side and the demand side. Therefore, to obtain robust estimates of activity they choose Industrial Production Index (supply side), total sales (demand side), real personal income less transfer payments (income side) and they add an employment variable to capture the idea that productivity do not change dramatically from one period to the other. In addition, due to its importance in determining the state of the business cycle, we enlarge this model by including the GDP series.

Table 2 shows the description of the series used for each economy, the sample period available and the data source. As we can observe in the table, the data available have all the characteristics mentioned before, ragged ends and mixing frequencies in a multivariate framework. For the case of the Euro area, there are no income variables available so we use wages and salaries released by Eurostat. In addition, the employment series is not available
monthly but quarterly. Due to data availability, in Spain we use Large Firm Sales (from Spanish Revenue Service) instead of retail sales, Social Security Contributors instead of employment, and Salaries Paid (from Spanish Revenue Service) instead of income.

One final remark deserves some comments. In Spain some series are published monthly but refer to annual growth rates. To obtain comparable results, we transform all the monthly indicators into annual growth rates. Accordingly, the annual growth rates, $x_t$, can be expressed as the sum of lagged monthly underlying variables:

$$x_t = \sum_{j=0}^{11} z_{t-j}. \tag{13}$$

### 3.3 Specification of the model

If we assume that all the variables are observed at monthly frequency, the model admits a simple state space representation.\(^8\) Let us assume that all the series in the model are AR(2), which implies that the monthly underlying series for quarterly observations are AR(6). Let $\sigma_f^2$, $\sigma_g^2$, and $\sigma_i^2$ be the variance of the common factor, the quarterly GDP growth rates and the monthly indicators, respectively. Let us use the following notation

\[ f_t = (f_t, f_{t-1}, \ldots, f_{t-11})', \tag{14} \]
\[ u_{xt} = (u_{gt}, u_{gt-6}, u_{gt-1}, u_{gt-2}, u_{gt-2}, u_{gt-2}, u_{gt-2}, u_{gt-2})', \tag{15} \]
\[ h_t = (f_{xt}', u_{xt}')', \tag{16} \]
\[ a_{xt} = (a_t, 0, \ldots, 0), \tag{17} \]
\[ \epsilon_{xt} = (\epsilon_g, 0, \ldots, 0, \epsilon_{gt}, 0, \epsilon_{gt}, 0, \epsilon_{gt}, 0, \epsilon_{gt}, 0), \tag{18} \]
\[ V_t = (a_{xt}', \epsilon_{xt}')'. \tag{19} \]

Hence, one can state the measurement equation as

$$ x_t = \begin{bmatrix} H \\ N \times 1 \end{bmatrix} \begin{bmatrix} h_t \\ N \times k \end{bmatrix} k \times 1, \tag{20} $$

and the transition equation as

$$ h_t = \begin{bmatrix} \mu_{si} + F \end{bmatrix} h_{t-1} + V_t, \tag{21} $$

\(^8\)We refer the reader to Camacho at al. (2009) to find the slight modification of the state space representation that allows for unobserved observations.
where $V_t$ is a multivariate white noise $(0, \mathbf{Q})$ with $\mathbf{Q}$ diagonal. In the measurement equation, the matrices are

$$
H = (H_1 \ H_2),
$$

where

$$
H_1 = \begin{pmatrix}
\frac{\lambda_1}{3} & 2\frac{\lambda_1}{3} & \lambda_1 & 2\frac{\lambda_1}{3} & \lambda_1/3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\lambda_2 & \lambda_2 & \lambda_2 & \lambda_2 & \lambda_2 & \lambda_2 & \lambda_2 & \lambda_2 & \lambda_2 & \lambda_2 & \lambda_2 & \lambda_2 & \lambda_2 & \lambda_2 \\
\lambda_3 & \lambda_3 & \lambda_3 & \lambda_3 & \lambda_3 & \lambda_3 & \lambda_3 & \lambda_3 & \lambda_3 & \lambda_3 & \lambda_3 & \lambda_3 & \lambda_3 & \lambda_3 \\
\lambda_4 & \lambda_4 & \lambda_4 & \lambda_4 & \lambda_4 & \lambda_4 & \lambda_4 & \lambda_4 & \lambda_4 & \lambda_4 & \lambda_4 & \lambda_4 & \lambda_4 & \lambda_4 \\
\lambda_5 & \lambda_5 & \lambda_5 & \lambda_5 & \lambda_5 & \lambda_5 & \lambda_5 & \lambda_5 & \lambda_5 & \lambda_5 & \lambda_5 & \lambda_5 & \lambda_5 & \lambda_5
\end{pmatrix},
$$

$$
H_2 = \begin{pmatrix}
1/3 & 2/3 & 1 & 2/3 & 1/3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0
\end{pmatrix},
$$

In the transition equation, the matrices are

$$
\mu_{st} = (\mu_{st}, 0,...0)',
$$

$$
\mathbf{F}_x = 
\begin{pmatrix}
0_{12 \times 12} & 0_{12 \times 14} \\
0_{14 \times 12} & F_{14 \times 14}
\end{pmatrix},
$$
Finally, the main diagonal of $Q$ is

$$(\sigma_f^2, 0, \ldots, 0, \sigma_g^2, 0, \ldots, 0, \sigma_1^2, 0, \sigma_2^2, 0, \sigma_3^2, 0, \sigma_4^2, 0)^\prime.$$  \hspace{1cm} (28)

### 3.4 Empirical results

Table 3 shows the maximum likelihood estimates of the more important coefficients for the three economies considered. As expected, GDP and to less extent IPI exhibit the higher loading factors. Although the magnitude of the other loading factors depend on the particular country, that of wages in the euro area is not significant probably because wages is a bad proxy of income variable. In addition, the mean of the common factor is positive in the first state and negative in the second state so we tentatively call them expansions and recessions. Finally, according to the estimates of $p_{ii}$, both recessions and expansions are very persistent.

Table 3 is complemented with figures 4, 5, 6 where we plot the estimated common factor $f_t$ for each economy and the filtered probabilities of recession periods. As mentioned before, the robustness of the results for the Euro area could be problematic so we left for
the next section a carefully treatment. In the case of US and Spain, the model gives a probability of being in a recession in September 2009 of 0.3 in the USA and 0.7 in Spain, which represents a significant improvement from the values of 0.98 that both economies exhibit in April-May.

According to these results, as of October 4th 2009, there is evidence to consider that the US economy is already out of the recession. However, it seems that the Spanish is still in its way to the end of the recession period. What it is clear from this exercise is that the timing of the signs of recovery is very distant from the timing about the explosion in the use of term "green shoots". According to Google trends, the maximum number of searches of this term is in the week of May 10th, and it is still extremely high until the week of the 24th of May. However, according to our results, the set of hard indicators that we use in the estimation of the probability of recession, still did not show up any real change in economic activity.\(^9\) Perhaps, the missing variable could be the expectations of the agents about the future of the economy.

### 3.5 Enlarging the original specification

As mentioned before, perhaps the solution could come from analyzing the role of the expectations. To account for expectations, we need to enlarge the model based on real time variables which mainly include expectation variables. In the related literature of small scale dynamic factor models, there are two linear proposal which incorporate indicators of expectations. The former is the so called Euro-Sting model by Camacho, and Perez Quiros (2009) and analyzes the Euro area. The latter is known as the Spain-Sting and it has been estimated for the Spanish economy.\(^10\) Both models have been designed for the linear framework and in our companion paper we enlarged the Euro-Sting to incorporate non linearities. In this paper, we also enlarge the Spain-Sting model to account for non linear dynamics.

\(^9\)In May 2009, Marcelle Chauvet still pointed out a probability of US recession of 0.77. See http://sites.google.com/site/marcellechauvet/probabilities-of-recession.

\(^10\)One interesting model for the US is the estimated by Aruoba, Diebold and Scotti (2009) which uses weekly variables. We still have not developed the non linear extension of this model, which is left for further research.
The Euro area model represents successive enlargements of the original model presented in the previous section. Starting from Stock and Watson (1991), we extend the model to capture the set of expectations indicators more promptly available in the Euro area.\textsuperscript{11} In that sense, we add Euro-zone Economic Sentiment Indicator (ESI), the German business climate index (IFO), the Belgian overall business indicator (BNB), and the Euro area Purchasing Managers confidence Indexes (PMI) in the services and manufacturing sectors. The main characteristics of these soft indicators are that they represent market expectations, and that they are promptly available so they can be observed on a timely basis within the reference month.

Once the model is enlarged with these indicators, we propose a method to decide whether new indicators should be added to this core. The method, which is based on the assumption that the primary focus of the model is to provide forecasts of GDP growth, consists of adding a variable only when it increases the percentage of variance of GDP growth explained by the common factor. With this method, we end up adding extra-Euro area exports and the Industrial New Orders index (INO, total manufacturing working on orders), to the set of core variables.\textsuperscript{12}

The model for the Spanish economy shares the same philosophy. It starts from the model estimated in the previous section, Industrial Production (excluding construction), total sales of large firms of Agencia Tributaria (Spanish Internal Revenue Service), social security contributors and total wages paid by the parge firms (Spanish Internal Revenue Service) as the set of core variables. The soft indicators used for the Spanish economy are the Product Manufacturing Index, and the Confident Index produced by the European Commission. But, in order to avoid overlapping in the information, from the supply side, we choose the Industrial Confident Indicator as an Index of the production sector, and the PMI services for the production of the services sector. From the demand side, we choose the Retail Trade Index.

We estimate this model, and the model fits the GDP data very precisely with a variance

\textsuperscript{11}We do not include Personal Disposable Income because we do not have this series for the euro area. As we showed in the previous section Salaries is a poor proxy variable for Income.

\textsuperscript{12}In order to address the process of data revisions in the GDP, we include flash and first GDP estimates as additional variables in the model.
of GDP explained by the common factor of 79%. However, given the knowledge that we have of the Spanish economy, and the fact that the model is very easily estimated, another possibility of enlargement is considered. The idea is, given the importance of the construction sector in explaining the recent boom and the crisis that we are currently living, to separate the supply sector in Industry, Services and Construction, choosing the most reliable series of each of them. The series suggested were, for Industry, the Industrial Production Series, for Services the Overnight Stays (Tourism represents more than 11% of Spanish GDP) and for Construction, Consumption of Cement. The demand side was enhanced with export and import series as indicator of external demand (as complementary of the internal demand captured by sales of large firms) and import series as indicator of demand that can not be satisfied with internal firms. Finally, a credit variable is added (total credit) in order to include a variable that captures the transmission of the financial crisis to the real economy.

In order to check if this enlargement is appropriate, we follow the criteria proposed in Camacho and Perez Quiros (2009). If the variance of GDP explained by the factor increases, this implies that the enlarged model fits better the data than the original one. In this case, the variance of GDP explained by the factor increases to 80%. Even though, this might sound as a minor increases, in this type of model is very frequent that, when adding variables correlated with the idiosyncratic part of some of the variables, the estimation of the factor is biased toward this subgroup, making the variance of GDP explained by the factor to decrease.

We allow in these two models for the possibility of a non-linear Markov switching specification. The results of the estimation are displayed in Table 4 and the filtered probabilities of being in a recession in every period of time are plotted in figures 6 and 7. According to the results, the expected duration of a recession in the Euro area and Spain is 15 and 13 months respectively. In this sense, if the current recession is coming to and end, and we situate the beginning of the recession period in the summer of 2008, this recession had a duration similar to the others. With respect to the amplitude, translating the negative growth associated to the recession period of -2.03 and -2.31 for the Euro area and the Spanish economy requires some algebra because these numbers
refer to a normalize monthly rates. The translation to a quarterly rate has to be done with the previously mentioned formula (10) which implies that the quarterly growth rate in a recession are -6.09 and -6.93 respectively. However in order to transform them into a expected quarterly growth rate in a recession period, we have to take into account that they are multiplied by 0.28 and 0.23 respectively and they have to be transformed with the standard deviation and the mean of the observed variables (0.68 and 0.57 for Spain). After these transformations, the expected growth rate in recessions is −.79 for the Euro area and −.22 for Spain. Therefore, the expected amplitude of the recessions would be around 3% in the Euro area, and 1% in the case of Spain much smaller, in the latter case, than the more than 4% already lost in the current recession.

With respect to the timing of the recession periods, figure 7 and 8 plot the filtered probabilities of being in a recession in the two economies considered. In the Euro area, once we incorporate the expectations, the probability of recession reaches to 0 in the recent months. In Spain, the probability of recession has been drastically reduced in the very recent months reaching in September 2009 only 30% which, technically could imply the end of the recession period.

3.6 Real time analysis

Even though we have been careful in presenting the filtered probabilities instead of presenting the smoothed probabilities, in order to make sure that we can address the probability of recession in every period of time with the information available until that period, the fact that we are only using available information in every period of time is not completely true. In the previous plots we use revised data as of today, but obviously, when making inference about the state of the economy in period t we only had available unrevised series. In addition, when estimating the filtering probabilities in T for all the previous periods, we are assigning information to period t, let’s say, the data for industrial production index on that period, that we are considering as part of the information set in t but was not available at that time.

In order to address the real flow of information, in order to analyze when we could call the end of the recession, we do a real time analysis exercise by estimating the model
in each period of time with the actual information available on each day. Figure 9 plots
the probability of recession in every period of time in the Euro Area with the information
available in every period. As it can be seen, in mid-july 2008 the probability of recession
increased up to be very close to one. At that time, we have to remember that the latest
GDP published was for the first quarter of 2008 and it was still a positive and very
high number (0.78). When the recession could be formally called (let’s use the classical
formulation of two quarters of negative GDP) it was when the data corresponding to the
third quarter of 2008 was published on November 15th 2008.

The second graph of the figure is the formal statistical formulation of the presence
of ”green shoots”. As it can be seen, at the end of April or the beginning of May there
were clear indications of recovery in the Euro with a probability of recession that went
drastically to cero at that time. In this case, we had evidence to announce the end of
the recession while still the latest published number of GDP growth was very negative (-2.45% for the first quarter of 2009).

It is important to understand how the mechanics of these signals works in order to
understand the reasons behind the change in probabilities. If we compare the value of
the soft indicators in the middle of April, when the probability of recession was still
very high, with the most recent realization of these indicators we can see that there is a
systematic positive trend. ESI, for example went from 64.6 to 82.8, PMIM from 33.9 to
49.3, etc...which, at the beginning (April, May) marked the turning point, but, which later
have been confirmed with a dramatic improvement in the value of the hard indicators IPI,
Retail Sales, INO and exports have changed from -2.38,-.78, -3.36 or -10.7 to values of
-28, -.21, +2.63 and 4.06 respectively.

The results for the Spanish economy have the same pattern but with a few months
delay. They are displayed in figure 10. First, as in the case of the Euro area we could
call a recession already by mid-february of 2008, while still the latest number for GDP
published for 2007.4 was giving an acceleration of the economy from 0.7 the third quarter
of 2007 to 0.8 the unrevised data for the fourth quarter of 2007. Again, as in the Euro
area case, the first numbers that gave an early signal were the soft indicators, Industrial
Confidence Index, PMIS, Retail Trade index that went down in two months, in the case
of the Industrial Confidence Index, from -4.2 to -9.3, in the case of the Retail Trade index from -13.1 to -26.3, or in the case of the PMIS from 51 to 46, for some of them, the biggest drop in the history of the indicator. The rest of the story is already well known with the deterioration in the hard indicators and the peak in the probabilities of recession.

With respect to the recovery, the second panel of the figure shows that it was after the summer 2009 when the economy showed signals of a turning point. Again, as in the Euro area case, this change in the probability of recession is at first driven by the latest release of soft indicators available (PMI services coming up by 5 points, from 40.8 to 45.3 from August to September, ICI coming from the historical low record of -40 in March to a reasonable -28 in September and Retail Trade Index, also coming from -29.4 to -21.8. These latest releases available are, in most cases, similar to the ones in the beginning of 2008 and then, compatible with positive growth rates of GDP.

However, these better realizations of the soft indicators still need to be confirmed by the hard indicator releases, and these are not available yet. We have no reason to believe that these indicators may give a false signal, because they have always gave the right signal in the past. However, there is still uncertainty at this point, and in order to better address the situation of the Spanish economy we should need to wait until some of the hard indicators are released.

4 The shape of the recovery. Euro area and Spain

Luckily, there is no a long history of recessions in these economies to make inference about what is the shape of the business cycle in these two regions. However, there is a long debate in the US on whether it will be a “V-shaped” or a “L-shaped” recession. The former type of recessions refers to the case in which the economy springs back rapidly from its slump whereas the latter type refers to the case in which the economy faces a period of flat or at best slowly improving performance. The economic implication of facing each of these types of recoveries is evident. The V-shaped recessions are viewed as evidence in favor of the Friedman’s plucking model, in which output cannot exceed a ceiling level but it is occasionally plucked downward by recessions which have only temporary
effects. On the contrary, recessions which are followed by flat recoveries are viewed as having permanent effects on the level of production. The paper by Camacho, Perez Quiros and Rodriguez (2009) makes a clear statement about what can we expect. According to these authors, the existence of the high recovery phase of the business cycle could be linked to inventory accumulation. These authors, citing previous literature postulate that until the early eighties firms maintain inventories to avoid stockouts. Under this theory, inventories reduce the opportunity costs of not servicing demand even at the cost of paying a price for holding the inventories which increases the volatility of output above that of sales. In periods of low demand inventories are low because the probability of stockout is low. As the economy exits the recessionary state, firms increase production not only to satisfy a growing demand but also to replenish inventories above the level they had during the recession. Accordingly, this process would explain the coexistence of periods of relatively high growth of production above the growth of sales right at the beginning of the expansions together with periods of high volatility. However, after the mid-eighties, the rapid improvements in information technology have led firms to reduce the production time and then rationalize the use of inventories. In that sense, firms do not need anymore to accumulate inventories because they are able to produce in real time exactly the amount they need to satisfy their sales without the risk of not servicing demand.

These results are very clear for the US, where the amount of recession periods allows the authors to make some inference about the path of the recoveries. One idea would be to consider that these results hold for the case of other economies. However this is not necessarily true. With respect to all major non-US economies, the paper by Camacho, Perez Quiros and Saiz (2008) make a clear contribution to understanding these facts. Based on industrial production series and with a bootstrapping methodology that allows them to compensate for the lack of recession periods, they find that the results for the US on the disappearance of the third-phase of the economy, associated with the accumulation of inventories and the V-shaped recessions, also hold for all major industrialized economies.
5 Conclusions

We find clear symptoms of recovery in the three economies analyzed, US, Euro area and Spain, with some differences in the timing and clarity of those symptoms. We focus on the differences between the real time inference between Spain and the Euro area, finding that the Euro area economy leads the Spanish economy in the appearance of the famous green shoots for which we are able to formulate a formal statistical definition.
References


Table 1. Markov-switching estimates

<table>
<thead>
<tr>
<th>Sample</th>
<th>including the latest recession</th>
<th>excluding the latest recession</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\mu_0$</td>
<td>$\mu_1$</td>
</tr>
<tr>
<td>US</td>
<td></td>
<td></td>
</tr>
<tr>
<td>53.1</td>
<td>1.02</td>
<td>-0.42</td>
</tr>
<tr>
<td>09.2</td>
<td>(0.08)</td>
<td>(0.26)</td>
</tr>
<tr>
<td>74.1</td>
<td>0.92</td>
<td>-0.53</td>
</tr>
<tr>
<td>09.2</td>
<td>(0.07)</td>
<td>(0.27)</td>
</tr>
<tr>
<td>91.3</td>
<td>0.76</td>
<td>-0.92</td>
</tr>
<tr>
<td>09.2</td>
<td>(0.07)</td>
<td>(0.26)</td>
</tr>
<tr>
<td>Euro area</td>
<td></td>
<td></td>
</tr>
<tr>
<td>91.3</td>
<td>0.56</td>
<td>-0.77</td>
</tr>
<tr>
<td>09.2</td>
<td>(0.05)</td>
<td>(0.15)</td>
</tr>
<tr>
<td>Spain</td>
<td></td>
<td></td>
</tr>
<tr>
<td>74.1</td>
<td>0.91</td>
<td>-0.08</td>
</tr>
<tr>
<td>09.2</td>
<td>(0.05)</td>
<td>(0.12)</td>
</tr>
<tr>
<td>91.3</td>
<td>0.84</td>
<td>-0.74</td>
</tr>
<tr>
<td>09.2</td>
<td>(0.05)</td>
<td>(0.12)</td>
</tr>
</tbody>
</table>

Notes. The estimated model is $y_t = \mu_s + \epsilon_t$, where $y_t$ is GDP growth rate, $\epsilon_t \sim iidN(0, \sigma^2)$, and $p(s_t = i | s_{t-1} = j) = p_{ji}$. 


Table 2. Indicators used in (5 series) Markov-switching factor model

<table>
<thead>
<tr>
<th>Series</th>
<th>Sample</th>
<th>Source</th>
<th>Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>US</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Industrial production</td>
<td>60.01-09.08</td>
<td>Datastream monthly</td>
<td>monthly</td>
</tr>
<tr>
<td>Retail sales</td>
<td>60.01-09.08</td>
<td>Datastream monthly</td>
<td>monthly</td>
</tr>
<tr>
<td>Employees in non farm</td>
<td>60.01-09.09</td>
<td>Bureau of Labor Statistics</td>
<td>monthly</td>
</tr>
<tr>
<td>Personal income less transfer payments</td>
<td>60.01-09.08</td>
<td>Datastream monthly</td>
<td>monthly</td>
</tr>
<tr>
<td>GDP</td>
<td>60.I-09.II</td>
<td>St. Louis FRED quarterly</td>
<td>quarterly</td>
</tr>
<tr>
<td>Euro area</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Industrial production</td>
<td>90.01-09.07</td>
<td>Eurostat monthly</td>
<td>monthly</td>
</tr>
<tr>
<td>Retail sales</td>
<td>95.01-09.07</td>
<td>Eurostat monthly</td>
<td>monthly</td>
</tr>
<tr>
<td>Employment</td>
<td>91.I-09.II</td>
<td>Eurostat quarterly</td>
<td>quarterly</td>
</tr>
<tr>
<td>Wages</td>
<td>95.01-08.12</td>
<td>Eurostat monthly</td>
<td>monthly</td>
</tr>
<tr>
<td>GDP</td>
<td>90.I-09.II</td>
<td>Eurostat quarterly</td>
<td>quarterly</td>
</tr>
<tr>
<td>Spain</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Industrial Production</td>
<td>83.01-09.08</td>
<td>INE</td>
<td>monthly</td>
</tr>
<tr>
<td>Large firm sales</td>
<td>95.01-09.08</td>
<td>Agencia Tributaria</td>
<td>monthly</td>
</tr>
<tr>
<td>Employment</td>
<td>83.01-09.09</td>
<td>Ministerio de Trabajo</td>
<td>monthly</td>
</tr>
<tr>
<td>Wages paid by large firms</td>
<td>95.01-08.12</td>
<td>Agencia Tributaria</td>
<td>monthly</td>
</tr>
<tr>
<td>GDP</td>
<td>83.1-09.II</td>
<td>INE</td>
<td>quarterly</td>
</tr>
</tbody>
</table>

Notes. To describe the sample, first two digits refer to the year, and last (two) digit(s) refers to the (month) quarter.
Table 3. (Main) Parameter estimates from (5 series) Markov-switching factor model

<table>
<thead>
<tr>
<th>Loading factors</th>
<th>GDP</th>
<th>Income</th>
<th>Sales</th>
<th>IP</th>
<th>Employ</th>
<th>Markov-switching parameters</th>
<th>μ₀</th>
<th>μ₁</th>
<th>p₀₀</th>
<th>p₁₁</th>
</tr>
</thead>
<tbody>
<tr>
<td>US</td>
<td>0.36</td>
<td>0.07</td>
<td>0.08</td>
<td>0.16</td>
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<td>0.22</td>
<td>-1.11</td>
<td>0.98</td>
<td>0.93</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.02)</td>
<td>(0.01)</td>
<td>(0.01)</td>
<td>(0.01)</td>
<td>(0.01)</td>
<td>(0.06)</td>
<td>(0.13)</td>
<td>(0.01)</td>
<td>(0.03)</td>
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</tr>
<tr>
<td>Euro area</td>
<td>0.49</td>
<td>0.006</td>
<td>0.08</td>
<td>0.18</td>
<td>0.03</td>
<td>0.08</td>
<td>-2.03</td>
<td>0.99</td>
<td>0.91</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.04)</td>
<td>(0.03)</td>
<td>(0.03)</td>
<td>(0.01)</td>
<td>(0.01)</td>
<td>(0.08)</td>
<td>(0.47)</td>
<td>(0.01)</td>
<td>(0.13)</td>
<td></td>
</tr>
<tr>
<td>Spain</td>
<td>0.26</td>
<td>0.08</td>
<td>0.08</td>
<td>0.11</td>
<td>0.06</td>
<td>0.14</td>
<td>-2.06</td>
<td>0.99</td>
<td>0.94</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.04)</td>
<td>(0.02)</td>
<td>(0.01)</td>
<td>(0.01)</td>
<td>(0.01)</td>
<td>(0.07)</td>
<td>(0.28)</td>
<td>(0.01)</td>
<td>(0.08)</td>
<td></td>
</tr>
</tbody>
</table>

Notes. The factor loadings (standard errors are in parentheses) measure the correlation between the common factor and each of the indicators appearing in columns. See Table 2 for a description of the indicators.
Table 4. (Main) Parameter estimates from Markov-switching extensions of Suro-STING and Spain-STING

<table>
<thead>
<tr>
<th>Indicator</th>
<th>Euro area Estimates (Standard deviations)</th>
<th>Spain Estimates (Standard deviations)</th>
</tr>
</thead>
<tbody>
<tr>
<td>GDP</td>
<td>0.2857 (0.034) GDP</td>
<td>0.236 (0.044)</td>
</tr>
<tr>
<td>IPI</td>
<td>0.3613 (0.045) IPI</td>
<td>0.073 (0.018)</td>
</tr>
<tr>
<td>Sales</td>
<td>0.1011 (0.033) Sales</td>
<td>0.083 (0.014)</td>
</tr>
<tr>
<td>INO</td>
<td>0.3312 (0.047) INO</td>
<td>0.095 (0.009)</td>
</tr>
<tr>
<td>Export</td>
<td>0.2022 (0.052) Export</td>
<td>0.062 (0.003)</td>
</tr>
<tr>
<td>ESI</td>
<td>0.078 (0.010) ESI</td>
<td>0.069 (0.016)</td>
</tr>
<tr>
<td>BNB</td>
<td>0.0999 (0.023) BNB</td>
<td>0.091 (0.012)</td>
</tr>
<tr>
<td>IFO</td>
<td>0.0837 (0.012) IFO</td>
<td>0.055 (0.020)</td>
</tr>
<tr>
<td>PMIM</td>
<td>0.1133 (0.014) PMIM</td>
<td>0.076 (0.012)</td>
</tr>
<tr>
<td>PMIS</td>
<td>0.1013 (0.018) PMIS</td>
<td>0.018 (0.007)</td>
</tr>
<tr>
<td>Employ</td>
<td>0.1251 (0.037) Employ</td>
<td>0.061 (0.009)</td>
</tr>
<tr>
<td>Ret. Trade Index</td>
<td>0.026 (0.019)</td>
<td>PMIS 0.047 (0.019)</td>
</tr>
</tbody>
</table>

Markov-switching parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Euro area Estimates (Standard deviations)</th>
<th>Spain Estimates (Standard deviations)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\mu_0$</td>
<td>0.37 (0.11) $\mu_0$</td>
<td>0.22 (0.09)</td>
</tr>
<tr>
<td>$\mu_1$</td>
<td>-2.04 (0.38) $\mu_1$</td>
<td>-2.31 (0.31)</td>
</tr>
<tr>
<td>$p_{00}$</td>
<td>0.97 (0.02) $p_{00}$</td>
<td>0.99 (0.01)</td>
</tr>
<tr>
<td>$p_{11}$</td>
<td>0.93 (0.06) $p_{11}$</td>
<td>0.92 (0.09)</td>
</tr>
</tbody>
</table>

Notes. First block refers to factor loading parameters. Second block refers to within expansion means ($\mu_0$), within recession means ($\mu_1$), and probabilities of staying in expansions ($p_{00}$) and recessions ($p_{11}$).
Figure 1. Growth rates of GDP

USA: GDP growth rates 1953.1-2009.2

Euro area. GDP growth rates 1993.1-2009.2

Spain. GDP growth rates 1970.1-2009.2

Note: Shaded areas correspond to recessions as documented by the NBER.
Figure 2. Filtered probabilities of recessions from GDP growth rates

Note: Shaded areas correspond to recessions as documented by the NBER.
Figure 3: Two periods ahead filtered probabilities of recession from GDP growth rates

Note: Shaded areas correspond to recessions as documented by the NBER.
Figure 4: US filtered probabilities of recessions and common factor from Markov-switching multivariate model (5 variables)

Filtered probabilities

![Filtered probabilities graph]

Common factor

![Common factor graph]

Note: Shaded areas correspond to recessions as documented by the NBER.
Figure 5: Euro area filtered probabilities of recessions and common factor from Markov-switching multivariate model (5 variables)
Figure 6: Spanish filtered probabilities of recessions and common factor from Markov-switching multivariate model (5 variables)

Note: Shaded areas correspond to recessions as documented by the ECRI.
Figure 7: Euro area filtered probabilities of recessions and common factor from Markov-switching Euro-STING model
Figure 8: Spanish filtered probabilities of recessions and common factor from Markov-switching Spain-STING model

Note: Shaded areas correspond to recessions as documented by the ECRI.
Figure 9: Real-time daily Euro area filtered probabilities of recessions and common factor from Markov-switching Euro-STING model.
Figure 10: Real-time daily Spanish filtered probabilities of recessions and common factor from Markov-switching Spain-STING model